Abstract

The essential norm of maximal and potential operators defined on homogeneous groups is estimated in terms of weights. The same problem is discussed for partial sums of Fourier series, Poisson integrals and Sobolev embeddings. In some cases we conclude that there is no a weight pair (v, w) for which the given operator is compact from L^p_w to L^q_v.

It is proved that the measure of non-compactness of a bounded linear operator from a Banach space into a weighted Lebesgue space with variable parameter is equal to the distance between this operator and the class of finite rank operators. The essential norm of the Hilbert transform acting from $L^{p(x)}_w$ to $L^{p(x)}_v$ is estimated from below. As a corollary we have that there is no a weight pair (v, w) and a function p from the class of log-Hölder continuity such that the Hilbert transform is compact from $L^{p(x)}_w$ to $L^{p(x)}_v$.

Necessary and sufficient conditions on a weight pair (v, w) governing the boundedness of generalized fractional maximal functions and potentials on the half-space from $L^p_w(\mathbb{R}^n)$ to $L^{q(x)}_v(\mathbb{R}^{n+1}_+)$ are derived. As a corollary, we have criteria for the trace inequality for these operators in variable exponent Lebesgue spaces.