

Abstract

Higher-order numerical techniques are developed for the solution of

(i) homogeneous heat equation $u_t = u_{xx}$

and

(ii) inhomogeneous heat equation $u_t = u_{xx} + s(x, t)$

subject to initial condition $u(x, 0) = f(x)$, $0 < x < 1$, boundary condition $u(0, t) = g(t)$ $0 < t \leq T$ and with non-local boundary condition(s)

(i) $\int_0^b u(x, t) dx = M(t)$ $0 < t \leq T$, $0 < b < 1$

(ii) $u(0, t) = \int_0^1 \phi(x, t) u(x, t) dx + g_1(t)$, $0 < t \leq T$ and

(iii) $u(1, t) = \int_0^1 \psi(x, t) u(x, t) dx + g_2(t)$, $0 < t \leq T$

as appropriate.

The integral conditions are approximated using Simpson's $\frac{1}{3}$ rule while the space derivatives are approximated by higher-order finite difference approximations. Then method of lines, semidiscretization approach, is used to transform the model partial differential equations into systems of first-order linear ordinary differential equations whose solutions satisfy recurrence relations involving matrix exponential functions. The methods are higher-order accurate in space and time and do not require the use of complex arithmetic. Parallel algorithms are also developed and implemented on several problems from literature and are found to be highly accurate. Solutions of these problems are compared with the exact solutions and the solutions obtained by alternative techniques where available.