

Abstract

Magic squares are among the most popular mathematical recreations. Over the years a number of generalizations have been proposed. In the early 1960's Sedlaek asked whether "Magic" ideas could be applied to graphs?

Shortly afterwards, in 1970 Kotzig and Rosa [defined a magic labeling of a graph. A labeling (or valuation) of a graph is a map that carries the graph elements to numbers (usually to the positive integers). The domain will usually be the set of all vertices and edges; such labelings are called total labelings. In the case when all vertices receive smallest labels, then edge-magic total (vertex-magic total) labeling is called super edge-magic total labeling (super vertex-magic total labeling).

Some labelings use the vertex set or the edge set alone and they will be referred as vertex-labeling and edge-labeling respectively. Other domains are also possible. There are many types of labelings for example harmonious, cordial, graceful and antimagic.

Enomoto et al. [conjectured that every tree admits a super edge-magic total labeling. In the effort of attacking this conjecture, many authors have considered super edge-magic total labeling for some particular classes of trees. Lee and Shah [have verified this conjecture for trees on at most 17 vertices with a computer help. Earlier, in [Kotzig and Rosa proved that every caterpillar is super edge magic. Banana tree and Lobster are particular classes of trees and this conjecture is still remains open for these classes.

Baëa et al.. [introduced the notion of a (a, d) -vertex-antimagic total labeling in 2000. In the case when all vertices receive smallest labels, (a, d) -vertex-antimagic total labeling is called super (a, d) -vertex-antimagic total labeling. Simanjuntak, Bertault, and Miller [define an (a, d) -edge-antimagic total labeling. In the case when all vertices receive smallest labels, (a, d) -edge-antimagic total labeling is called a snper (a, d) -edge-antimagic total labeling.

Super (a, d) -edge-antimagic total labeling of cycles with chords (Harary graph) has been studied by M. Baëa and M. Murugan [6]. They conjectured that:

There is a super $(2n + 2, 1)$ edge-antimagic total labeling of Harary graph for (i) $n \equiv 0 \pmod{4}$ and $t \equiv 0 \pmod{4}$, and

(ii) $n \equiv 2 \pmod{4}$ and for t even.

This dissertation studies about the construction of a super edge-magic total labeling of banana tree as well as for disjoint union of k identical copies of banana trees. We also construct the super (a, d) -edge-antimagic total labeling and super (a, d) -vertex-antimagic total labeling on Harary graph as well as for disjoint union of k identical copies of Harary graphs.

This dissertation also studies about the construction of vertex-antimagic total labeling of Harary graph as well as for disjoint union of k identical copies of the Harary graphs and a super vertex-magic total labeling of Harary graph.