Abstract

Deciding where to begin is a major step. One procedure is to lay out all necessary preliminary material, introduce the major ideas in their most general setting, prove the theorems and then specialize to obtain classical results and various applications. We experience convexity all the times and in many ways. The most prosaic example is our upright position, which is secured as long as the vertical projection of our center of gravity lies inside the convex envelope of our feet. Also convexity has a great impact on our every day life through numerous applications in industry, business, medicine and art. So do the problems of optimum allocation of resources and equilibrium of non cooperative games. The theory of convex functions is a part of the general subject of convexity, since a convex function is one whose epigraph is a convex set. Nonetheless it is an important theory, which touches almost all branches of mathematics.

In calculus, the mean value theorem states, roughly, that given a section of a smooth curve, there is a point on that section at which the derivative (slope) of the curve is equal (parallel) to the "average" derivative of the section. It is used to prove theorems that make global conclusions about a function on an interval starting from local hypotheses about derivatives at points of the interval. This theorem can be understood concretely by applying it to motion: if a car travels one hundred miles in one hour, so that its average speed during that time was 100 miles per hour, then at some time its instantaneous speed must have been exactly 100 miles per hour.

An early version of this theorem was first described by Parameshvara (1370-1460) from the Kerala school of astronomy and mathematics in his commentaries on Govindasvami and Bhaskara II. The mean value theorem in its modern form was later stated

by Augustin Louis Cauchy (1789-1857). It is one of the most important results in differential calculus, as well as one of the most important theorems in mathematical analysis, and is essential in proving the fundamental theorem of calculus. The mean value theorem can be used to prove Taylor's theorem, of which it is a special case.
We use this Mean value theorem and its other generalized version to define new Cauchy's means.
In the first chapter some basic notions and results from the theory of means and convex functions are being introduced along with classical results of convex functions.

In the second chapter we define some further results about logarithmic convexity of differences of of power means for positive linear functionals as well as some related results.
In the third chapter we define new means of Cauchy's type. We prove that this mean is monotonic. Also we give some applications of this means.
In the fourth chapter we give Cauchy's means of Boas type for non positive measure. We show that these Cauchy's means are monotonic.
In the fifth chapter, we give definition of Cauchy means of Mercer's type. Also, we show that these means are monotonic.
In the sixth chapter, we define the generalization of results given by S. Simić, for log-convexity for differences of mixed symmetric means. We also present related Cauchy's means.
In the last chapter we give an improvement and reversion of well known Ky-Fan inequality. Also we introduce in this chapter Levinson means of Cauchy's type. We prove that these means are monotonic.