

Abstract

In this thesis we give a structure theorem for Cohen-Macaulay monomial ideals of codimension 2, and describe all possible relation matrices of such ideals. We also study the set $\mathcal{T}(I)$ of all relation trees of a Cohen-Macaulay monomial ideal of codimension 2. We show that $\mathcal{T}(I)$ is the set of bases of a matroid. In case that the ideal has a linear resolution, the relation matrices can be identified with the spanning trees of a connected chordal graph with the property that each distinct pair of maximal cliques of the graph has at most one vertex in common.

We give the equivalent conditions for a squarefree monomial ideal to be a complete intersection. Then we study the set of Cohen-Macaulay monomial ideals with a given radical. Among this set of ideals are the so-called Cohen-Macaulay modifications. Not all Cohen-Macaulay squarefree monomial ideals admit nontrivial Cohen-Macaulay modifications. It is shown that if there exists one such modification, then there exist indeed infinitely many. We also present classes of Cohen-Macaulay squarefree monomial ideals with infinitely many nontrivial Cohen-Macaulay modifications.