Abstract

Since the Galois absolute group $G = Gal(\overline{\mathbb{Q}}/\mathbb{Q})$ does not act continuously on $\overline{\mathbb{Q}}$, the usual topology of $\mathbb{C}$ cannot be directly used to study the group G ($\overline{\mathbb{Q}}$ is dense in $\mathbb{C}$). In this thesis we consider a finite (or infinite) extension L of $\mathbb{Q}$, in $\overline{\mathbb{Q}}$ and its corresponding absolute group $G_L = Gal(\overline{\mathbb{Q}}/L)$. On $\overline{\mathbb{Q}}$ we introduce a norm,

$$x \to \|x\|_L = \max\{|\sigma(x)| : \sigma \in G_L\}$$

and call it the L-(or G_L-) spectral norm. Let $\widetilde{\mathbb{Q}}_L$ be the completion of $\overline{\mathbb{Q}}$ with respect to $\|.\|_L$. Since $\|\sigma(x)\|_L = \|x\|_L$ for any $\sigma \in G_L$, G_L acts continuously on $\overline{\mathbb{Q}}$. We prove many results on the structure of $\widetilde{\mathbb{Q}}_L$ and we connect it with G_L itself.

We also introduce the new notion of a v-adic maximal extension $L^{(v)}$ of a valued field (K, v) and we supply with some fundamental results relative to the structure of $L^{(v)}$. We also connect it with some particular types of spectral norms.

Some other auxiliary results (a strange functional generalization of Krasner's Lemma, for instance) which are useful by their own are given.