

Abstract

Necessary and sufficient conditions on a weight function v guaranteeing the boundedness/compactness of the Riemann-Liouville transform with variable parameter $R_{\alpha(x)}$ from $L^{p(x)}$ to $L_v^{q(x)}$ are found. The measure of non-compactness of this operation is also estimated from both sides in variable exponent spaces.

Necessary and sufficient conditions guaranteeing the trace inequality for positive kernel operators in classical Lebesgue spaces defined on cones of nilpotent groups are established. Compactness criteria for these operators in classical weighted Lebesgue spaces are also obtained. Two-sided estimate for Schatten-von Neumann ideal norms of weighted higher order kernel operators are established. Asymptotic formulas for singular numbers for some potential-type operators are derived. Some of these results are applied to the problem of the existence of non-negative solution for certain non-linear integral equation.